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MEMORANDUM REPORT NO. 1775

## NOTE ON SHOCK REFLECTION COEFFICIENT

by

Nathan Gerber

August 1966

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RDTE&E Project No. 1P222901A201

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NGerber/sjw  
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NOTE ON SHOCK REFLECTION COEFFICIENT

ABSTRACT

The shock reflection coefficient,  $R_w$ , is defined here as the ratio of pressure drop across a reflected disturbance from a shock wave to that across the incident disturbance wave (which is generated downstream of the shock). The treatment of shock reflection coefficients in the work of previous investigators is reviewed briefly. A derivation is given of a general expression for  $R_w$  in terms of shock curvature for nonequilibrium axisymmetric flow. Sample computations show the effects of nonequilibrium flow and three-dimensionality on the behavior of  $R_w$ . It is shown that  $R_w$  can become infinite in these situations, although for two-dimensional ideal gas flow its magnitude is always finite and usually very small.

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# LIST OF SYMBOLS

- f Nonequilibrium production term, Eq. (4)
- $K_w$  Shock wave curvature
- M Local Mach number
- n Arc length on curve normal to streamlines
- p Pressure
- q Flow speed
- $R_w$  Shock reflection coefficient
- s Arc length on streamline
- T Temperature
- x Coordinate in direction of free stream
- y Coordinate normal to free stream
- $\beta$  Shock wave angle
- $\gamma$  Ratio of specific heats
- $\epsilon$  = 0 for two-dimensional flow; = 1 for axisymmetric flow.
- $\eta$  Arc length on left running Mach line
- $\theta$  Direction of velocity vector
- $\lambda$  =  $\beta - \theta$
- $\mu$  Mach angle, =  $\arcsin(1/M)$
- $\xi$  Arc length on right running Mach line
- $\rho$  Density
- Subscripts:
- $\infty$  Free stream value
- w Value at shock wave

## INTRODUCTION

The shock reflection coefficient, designated here by  $R_w$ , is a quantity of interest in the study of interaction of shock and Mach waves, particularly in connection with application of the shock-expansion method to computing supersonic flows. Essentially it provides a measure of the strength of a reflected disturbance from a shock wave relative to the strength of the incident disturbance wave (which is generated downstream of the shock).

Lighthill<sup>1</sup>, Chu<sup>2</sup>, and Chernyi<sup>3\*</sup> have obtained the reflection coefficient for two-dimensional ideal gas flow. (See page 180 of Reference 3 for a brief discussion of errors in References 1 and 2.) Graphs of computations found in References 3 and 4 show that the magnitude of  $R_w$  is small except near the region of shock detachment, and that  $|R_w| < 1$  for finite Mach number. Note, however, that  $R_w = -1$  for a straight shock and finite flow deflection, according to the definition in Equation (1). Eggers and Syvertson<sup>5</sup> consider a closely related quantity called "disturbance strength ratio." Their calculation for a case of air in vibrational equilibrium ( $M_\infty = 10$ ,  $T_\infty = 278^\circ \text{ K}$ ) indicates that caloric imperfections in the gas still leave the disturbance strength ratio small compared to unity if  $\gamma$  does not decrease appreciably below 1.3.

We now wish to investigate nonequilibrium effects on the shock reflection coefficient; here the shock curvature appears explicitly. In addition, we wish to examine the effect of three-dimensionality, after extending the definition of  $R_w$  to axisymmetric flow.

## EQUATIONS

Refer to Figure 1; the shock reflection coefficient is defined as follows (as on page 232 of Reference 6):

$$R_w \equiv \lim_{\Delta \xi, \Delta \eta \rightarrow 0} \frac{p_C - p_B}{p_B - p_A} = \lim_{\Delta \xi, \Delta \eta \rightarrow 0} \frac{(dp/d\eta)_B \Delta \eta}{(dp/d\xi)_A \Delta \xi} \quad (1)$$

\* Superscript numbers denote references which may be found on page 15.

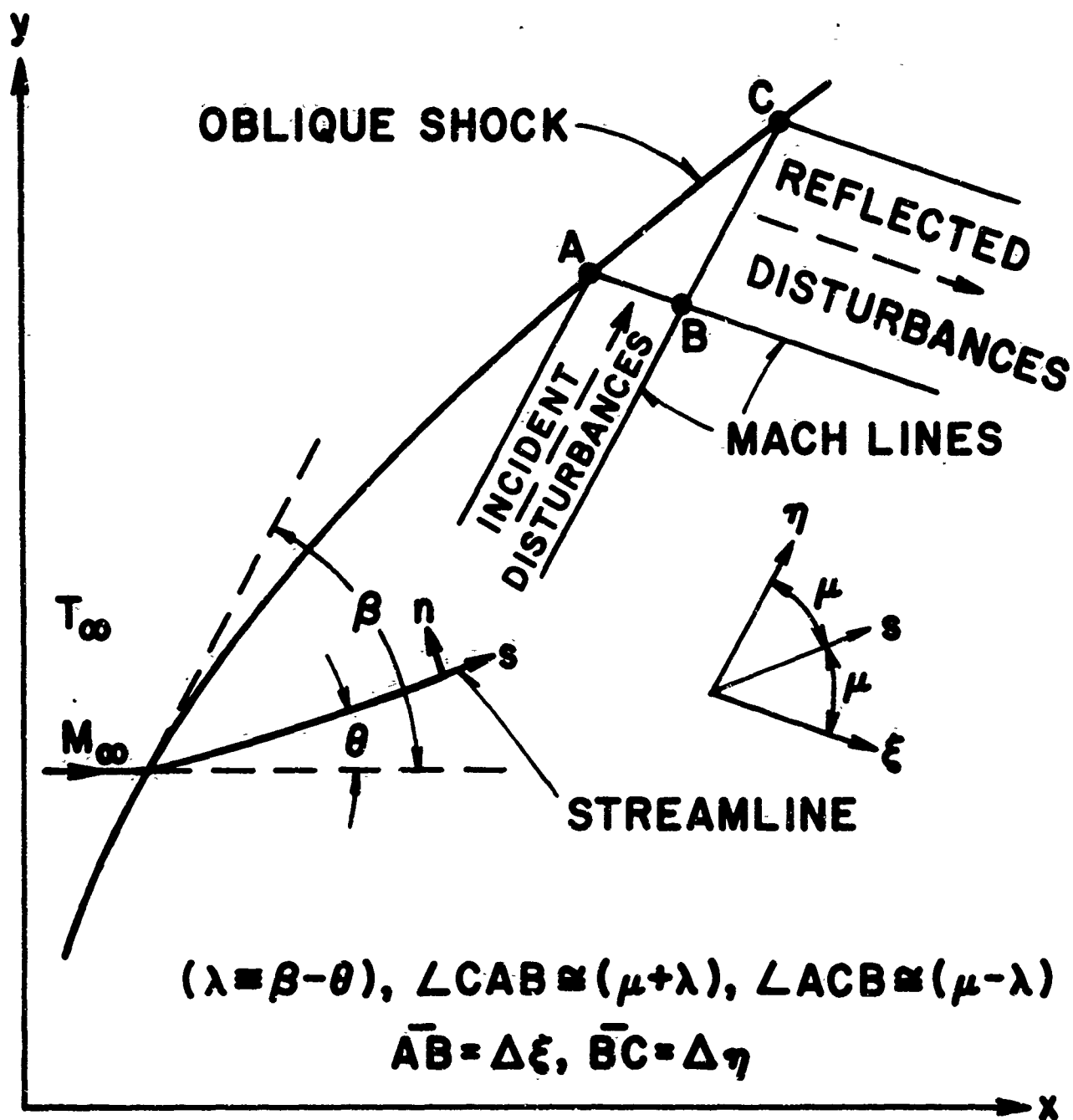


FIGURE 1. CROSS-SECTIONAL VIEW OF FLOW FIELD

Here the positive  $\xi$  and  $\eta$  directions make angles of  $\mu (< 90^\circ)$  with the positive streamline direction. Applying the law of sines to  $\Delta ABC$ , we get

$$R_w = \frac{(\partial p / \partial \eta)}{(\partial p / \partial \xi)} \frac{\sin(\mu + \lambda)}{\sin(\mu - \lambda)} \quad (2)$$

By means of the directional derivative relations

$$\partial p / \partial \eta = (\partial p / \partial s) \cos \mu + (\partial p / \partial n) \sin \mu$$

$$\partial p / \partial \xi = (\partial p / \partial s) \cos \mu - (\partial p / \partial n) \sin \mu$$

and the following relation valid at the shock wave:

$$(\partial / \partial n)_w = (\csc \lambda)_w [K_w \partial / \partial \beta - (\cos \lambda) \partial / \partial s]_w$$

the shock reflection coefficient is expressed as

$$R_w = - \frac{\sin(\mu + \lambda)}{\sin(\mu - \lambda)} \frac{\left[ (\sin \mu) \frac{\partial p}{\partial \beta} K_w - \frac{\partial p}{\partial s} \sin(\mu - \lambda) \right]}{\left[ (\sin \mu) \frac{\partial p}{\partial \beta} K_w - \frac{\partial p}{\partial s} \sin(\mu + \lambda) \right]} \quad (3)$$

and  $\partial p / \partial s$  is given by \*

$$D \frac{\partial p}{\partial s} = (K_w \csc \lambda) \left( \frac{\partial p}{\partial \beta} \cot \lambda + \rho q^2 \frac{\partial \theta}{\partial \beta} \right) + \frac{\epsilon}{y} \rho q^2 \sin \theta + \rho q^2 f \quad (4)$$

where  $D = 1 - M^2 + \cot^2 \lambda$ , and  $f$  is the nonequilibrium "production term" ( $f = 0$  for frozen or equilibrium flow);  $\epsilon = 0$  or  $1$  for two-dimensional or axisymmetric flow, respectively.

### CALCULATIONS

We consider dissociating air, using the model presented in Reference 7, as an example of nonequilibrium effects on  $R_w$ ; for this flow the air is in vibrational equilibrium at the shock. A typical set of results is shown in Figure 2 for a wide range of shock curvature. It is seen that for each value of  $K_w$  there is a critical deflection angle producing infinite  $R_w$ ; this results from the vanishing of  $\partial p / \partial \xi$  in Equation (2).

\* This formula is found in section 5 of Reference 7, where a typographical error occurs; the multiplication sign should be replaced by a plus sign.



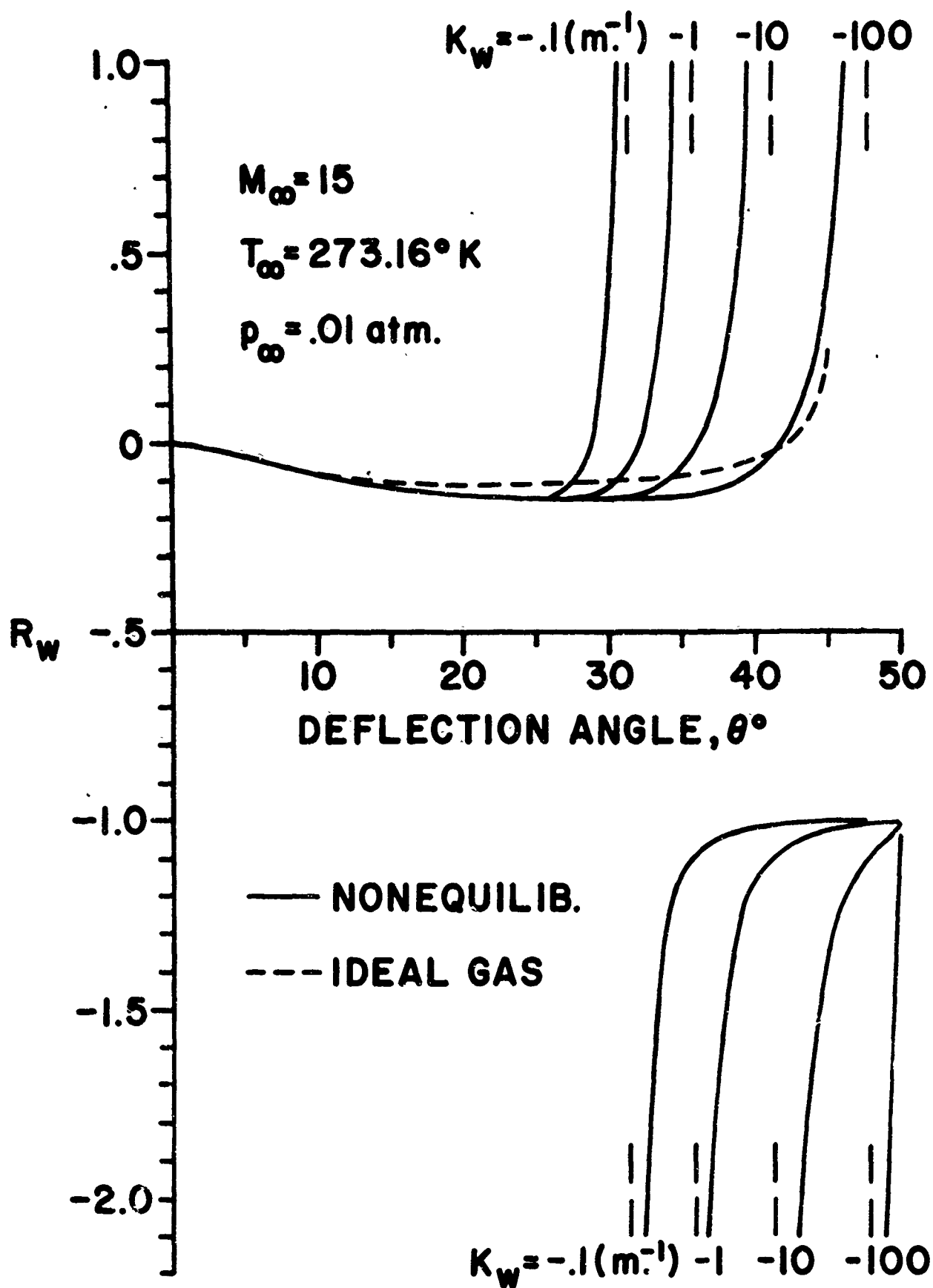


FIGURE 2. SHOCK REFLECTION COEFFICIENT FOR NONEQUILIBRIUM DISSOCIATING AIR, TWO DIMENSIONAL FLOW

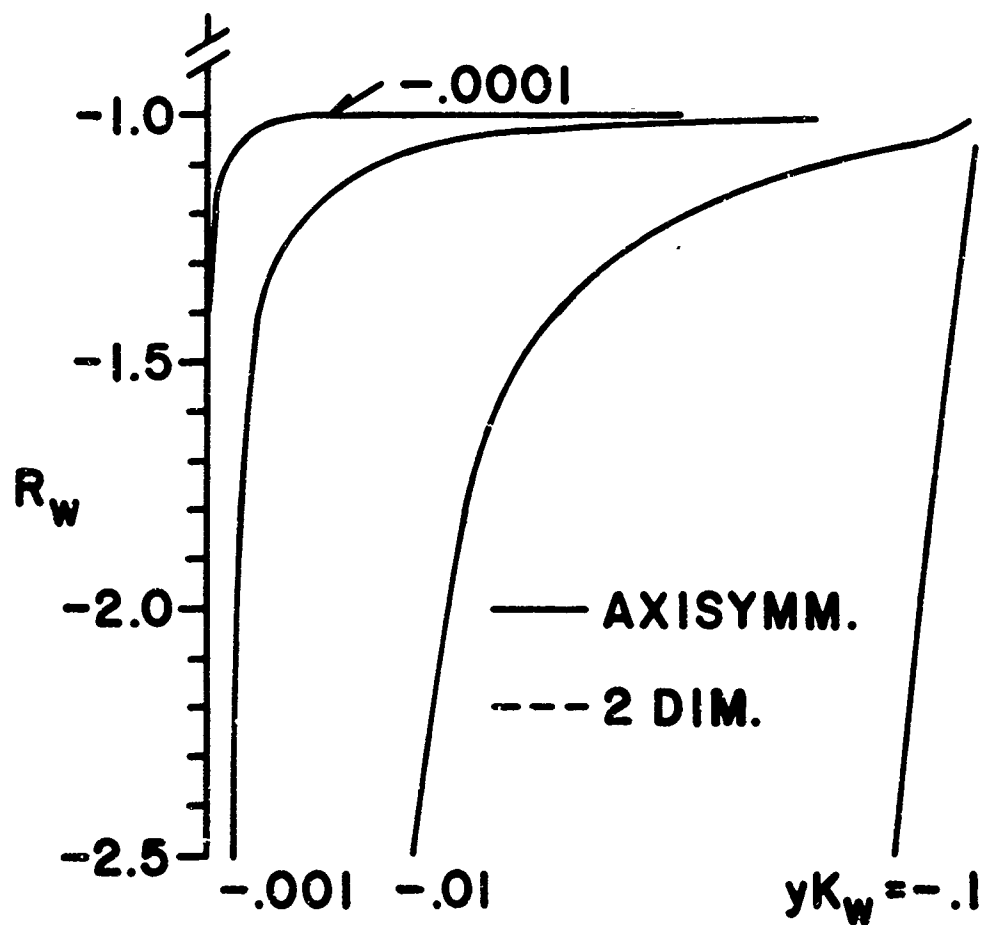
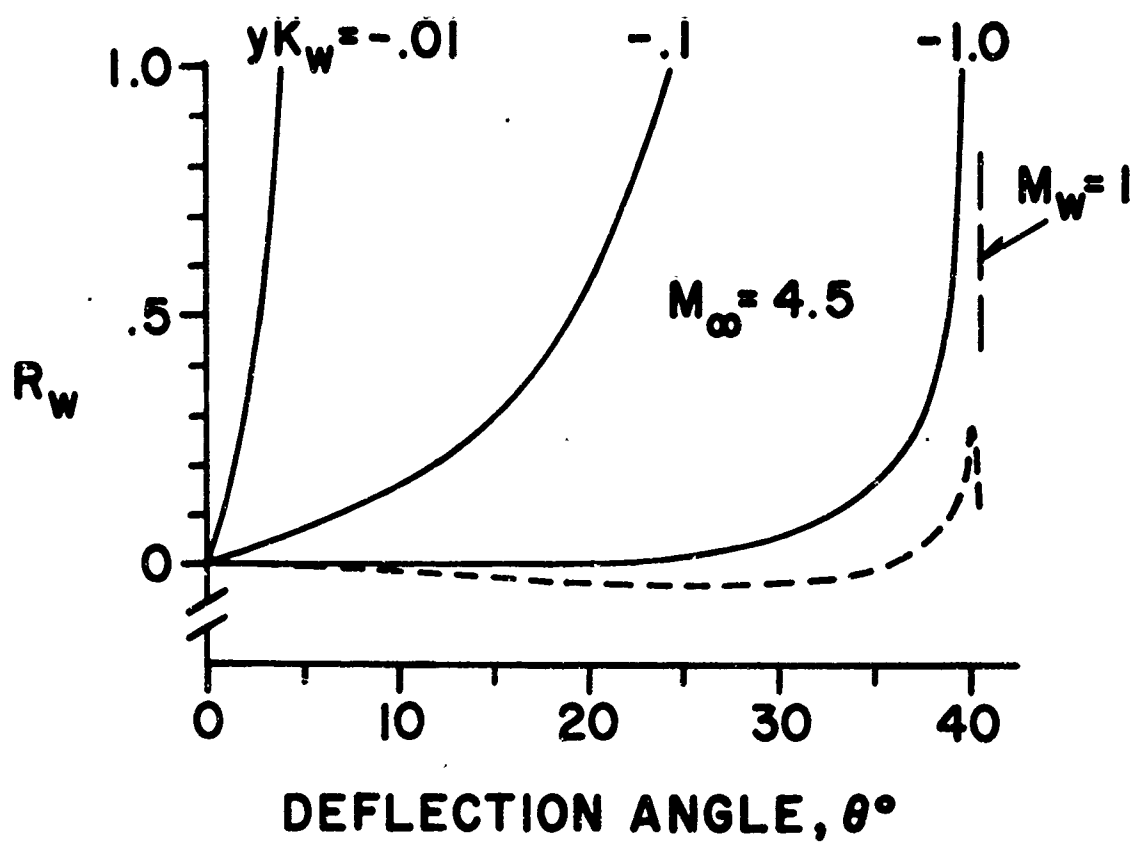


FIGURE 3. SHOCK REFLECTION COEFFICIENT FOR AXISYMMETRIC FLOW, IDEAL GAS

For deflections smaller than this critical value the behavior of  $R_w$  is similar to that for ideal gas flow except for the sharp rise at the asymptote; beyond the asymptote  $R_w$  approaches  $-1$  at the detachment angle.

When  $\epsilon$  is taken equal to 1 in Equation (4), the definition of shock reflection coefficient [(Equation (1))] can be applied to axisymmetric flow. Figure 3 shows some typical results in ideal gas for several values of the parameter  $yK_w$ . (Generally for flows over smooth ogival bodies of revolution,  $-0.1 < yK_w \leq 0$ .) The behavior of  $R_w$  here differs considerably from that for two-dimensional flow. There is a critical deflection angle for each  $yK_w$  and a limiting  $R_w$  value of  $-1$  as detachment angle is approached. Formally,  $yK_w = -\infty$  corresponds to two-dimensional flow, and the figure shows the approach of the curves with decreasing  $yK_w$  to the two-dimensional one.

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NATHAN GERBER

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